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ARAP REPORT NO. 132

AERODYNAMIC FORCES OF FLUTTERING CYLINDRICAL AND/OR PLANAR STRUCTURES

by

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Prepared under

Contract No. NAS8-21294 for

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

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PREFACE

This report covers work initiated by the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, Huntsville, Alabama under Contract NAS8-21294. The work is administered by Mr. R. G. Beranek.

This report covers work done under this contract for the period 13 February 1968 to 31 March 1969. The principal investigator of the program was Dr. E.F.E. Zeydel until his death 18 October 1968. From 1 December 1968 until 31 March 1969 the principal investigator was Dr. J. E. Yates.

The work reported herein is, with minor exceptions, that of Dr. Zeydel. The principal effort of Dr. Yates was to ascertain the status of Dr. Zeydel's work and prepare the final report.

The many helpful suggestions of Dr. J. C. Houbolt during the preparation of the final report are gratefully acknowledged.

ABSTRACT

The method introduced in Ref. 1 for calculating flutter boundaries of an infinite spanwise array of low aspect ratio panels is extended to include membrane stresses and the alternative of pinned or clamped leading and trailing edges. The technique is developed to the point where numerical calculations can be made. Examples are given for the pinned edge case. The rate of calculating flutter points is typically about 200 times greater than the Galerkin procedure for a panel length-to-width ratio of ten.

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LIST OF SYMBOLS

a_∞	= speed of sound in air
a_s	= speed of longitudinal waves in the panel, (defined by 3.2)
A	= defined by (2.7)
b	= span of the panel
B	= defined by (2.7)
D_o	= $Eh^3/12(1 - \nu^2)$, flexural rigidity
D	= $D_o\ (1 + ig)$
$\bar{D}(p)$	= defined by (2.12)
$D(k,s)$	= defined by (2.17) for pinned edges and (2.19) for clamped edges
D_R, D_I	= real and imaginary parts of $D(k,s)$
E	= Young's modulus
$F(x)$	= defined by (2.13) (Also see Appendix A)
g	= structural damping parameter
h	= panel thickness
i	= $\sqrt{-1}$
$J_o(z)$	= Bessel function of the first kind of order zero
k	= $\frac{\omega b}{U}$, reduced frequency
K	= $\frac{kM}{\beta^2}$
m	= number of half waves in the chordwise mode shape
M	= Mach number
$\bar{n}$	= number of half waves in the span b
N_x, N_y	= membrane stresses
p	= Laplace transform variable
$P(X,Y)$	= pressure amplitude
$P(X)$	= chordwise pressure function, defined by (2.8)
r_x, r_y	= dimensionless membrane stresses, defined by (2.3)
R	= defined by (2.3)
s	= length-to-width ratio
S	= defined by (2.3)

t	=	time
U	=	air velocity
w	=	panel deflection
W	=	downwash function, defined by (2.8)
x,y	=	streamwise and spanwise coordinates
β	=	$\sqrt{M^2 - 1}$
Γ	=	defined by (2.8)
δ,η,μ	=	defined by (3.1)
ν	=	Poisson's ratio
ρ_∞	=	air density
ρ_s	=	panel density
$\phi(x)$	=	chordwise mode shape
$\bar{\phi}(p)$	=	Laplace transform of $\phi(x)$
ω	=	flutter frequency

I. INTRODUCTION

In previous analyses of panel flutter, methods have been developed to obtain approximate solutions of the complete problem and exact solutions of various approximations of the problem. In the first instance the Galerkin or assumed mode approach has become more or less standard. On the other hand, the two approximations of the problem that have been given the most serious attention are those based on static aerodynamic theory and quasi-steady theory. Exact methods have been used to attack both of these approximations of the problem (e.g., see Refs. 2, 3, 4).

The approximations based on static and quasi-steady theory have served an important role in providing basic understanding of panel flutter phenomena, and indeed each theory has a significant domain of quantitative validity; namely, for low-frequency and high supersonic Mach number. Unfortunately, both approximations fail in the low supersonic regime which has been shown to be a critical region for panel flutter (Refs. 5, 6, 7). In this region one must, perforce, retain the exact aerodynamic forces in the formulation of the problem. It is at this point that previous investigators have had to resort to the modal approach and settle for approximate solutions.

The Galerkin method is a very powerful tool for solving the exact problem when the length-to-width ratio of the panel is of order unity or smaller. For such geometry the flutter mode shape can be approximated closely with only a few (usually two) natural vibration modes so that the resulting flutter matrix is of tractable size. However, when the length to width ratio becomes large the number of half waves in the chordwise flutter mode becomes large and also the mode shape has a strong exponential growth in the chordwise direction with largest deflection near the trailing edge. (See e.g. Refs. 3 and 4). To resolve this type of flutter mode into Fourier components, one must use a large number of vacuum normal modes and consequently must be able to handle a very large flutter matrix to obtain convergence (See Ref. 7). The result is that the computational effort required to obtain a single flutter point grows out of proportion to the magnitude of the problem.

In Ref. 1 a new method for solving an exact formulation of the problem was introduced. The problem treated is that of an infinite spanwise array of identical panels. (For a comparison of results for such an array with a discrete panel see Ref. 8). Exact inviscid aerodynamic theory is employed. The technique is to solve the problem by Laplace transforms in a way suggested by Goland and Luke, Ref. 9, for investigating the problem of membrane flutter. The difficulties associated with the Galerkin method are circumvented in that the mode shape and its derivatives are obtained quite naturally in the process of solution. The present work is an extension of Ref. 1 to include membrane stresses and the alternative of pinned or clamped leading and trailing edges. The pinned edge case has been developed to a point where numerical calculations can be performed. Example calculations are presented.

II. METHOD OF SOLUTION

The problem we consider is an infinite array of identical rectangular panels in a supersonic main stream of velocity U . All lengths are referred to the panel span, b , and the length-to-width ratio is denoted by s . We assume simple harmonic motion so that the panel deflection is of the form:

$$w = \mathrm{Re}\left(w(x,y)e^{i\omega t}\right) \tag{2.1}$$

where ω is the flutter frequency and w is the complex flutter amplitude. The equation of a typical panel that undergoes simple harmonic motion is (see, e.g., Ref. 1)

$$\nabla^4 w - r_x \frac{\partial^2 w}{\partial x^2} - r_y \frac{\partial^2 w}{\partial y^2} - Rk^2 w + S\,P(x,y) = 0 \tag{2.2}$$

where

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

$$R = \frac{\rho_s h b^2 U^2}{D} \quad , \quad S = \frac{\rho_\infty U^2 b^3}{D}$$

$$r_x = \frac{N_x b^2}{D} \quad , \quad r_y = \frac{N_y b^2}{D}$$

$$k = \frac{\omega b}{U} \tag{2.3}$$

and P(x,y) is the pressure. Structural damping is included in (2.2) through the flexural rigidity

$$D = D_o(1 + ig) \quad , D_o = Eh^3/12(1 - \nu^2) \tag{2.4}$$

where g is the usual structural damping parameter . Also, we note that N_x and N_y are midplane stresses in the chordwise and spanwise directions, respectively. Midplane shear is assumed to be negligible.

In the present study, we assume that the spanwise edges $(y = m\pi)$ are pinned and adopt a deflection function of the form

$$w(x,y) = f(x) \sin \bar{n}\pi y \tag{2.5}$$

where $\bar{n}$ is the number of half-waves across the span of the panel. Substitute (2.5) into (2.2) to obtain the equation that governs the chordwise modal function $f(x)$:

$$\frac{d^4 f}{dx^4} - A \frac{d^2 f}{dx^2} + Bf + S\,P(x) = 0 \tag{2.6}$$

where

$$A = 2(\bar{n}\pi)^2 + r_x$$

$$B = (\bar{n}\pi)^4 + (\bar{n}\pi)^2 r_y - Rk^2 \tag{2.7}$$

The chordwise aerodynamic pressure $P(X)$ is due to a line of supersonic sources across the span whose strength varies as $\sin \bar{n}\pi y$. From Ref. 1, we have

$$P(X) = \frac{1}{\beta}\int_0^x e^{-iKM(x - \xi)} J_o\left[\Gamma(x - \xi)\right]\left[W'(\xi) + ikW(\xi)\right]d\xi$$

$$+ \frac{e^{-iKMX}}{\beta} J_o(\Gamma x) f'(0) \tag{2.8}$$

where the downwash is given by

$$W(x) = f'(x) + ikf(x)$$

$$\Gamma = \left(K^2 + \frac{\bar{n}^2\pi^2}{\beta^2}\right)^{\frac{1}{2}} \quad , \quad K = \frac{kM}{\beta^2} \tag{2.8}$$

The boundary conditions at the leading and trailing edges are specified as follows:

$$\begin{aligned} f(0) &= f(s) = 0 \quad \text{pinned or clamped} \\ f''(0) &= f''(s) = 0 \quad \text{pinned} \\ f'(0) &= f'(s) = 0 \quad \text{clamped} \end{aligned} \tag{2.9}$$

The parametric domain of panel flutter is given by the solutions of (2.6) subject to the boundary conditions of pinned or clamped leading and trailing edges.

The usual procedure for solving the foregoing problem is the Ritz-Galerkin assumed mode approach. When the length-to-width ratio s of the panel is small, between zero and one, say, this technique is an effective tool for panel flutter calculations. However, when s becomes large, the natural frequencies of the panel become very closely spaced and many natural modes must be used to represent the flutter mode. To circumvent the difficulties associated with the Galerkin procedure, Dr. Zeydel had in recent years been developing a new technique (see Ref. 1) that is based on the Laplace transform. It yields directly the flutter boundary and mode shapes that are essential for stress calculations, without reference to assumed modes and the numerical problems associated with large matrices. The remainder of this section is devoted to a detailed explanation of this procedure.

Introduce the Laplace transform in the chordwise direction:

$$\bar{f}(p) = \int_0^\infty e^{-px} f(x)dx$$

$$f(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} e^{px} \bar{f}(p)dp \tag{2.10}$$

where γ is to the right of all singularities of $\bar{f}$. Now, take the Laplace transform of (2.6) to obtain

$$\bar{f}(p) = \frac{(p^2 - A) f_o' + pf_o'' + f_o'''}{\bar{D}^+(p)} \tag{2.11}$$

where

$$\bar{D}^+(p) = p^4 - Ap^2 + B + \frac{S}{\beta} \frac{(p + ik)^2}{\left[(p + iKM)^2 + r^2\right]^{\frac{1}{2}}} \tag{2.12}$$

The last term on the right-hand side of (2.12) is the Laplace transform of the pressure that is given by (2.8). The subscript o in (2.11) means $x = 0$ or leading edge. Also, we note that the boundary condition $f_c = 0$ has been used and that either f_o' or f_o'' is zero when the distinction of pinned or clamped leading edge is made. For the moment, we shall carry both terms in (2.11).

Next, we define the function

$$F(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{e^{px}}{\bar{D}^+(p)} dp \tag{2.13}$$

that plays a fundamental role in the present technique. We remark that $F(x)$ is a solution of (2.6) subject to the boundary conditions

$$f_o = f_o' = f_o'' = 0$$

$$f''' = 1 \tag{2.14}$$

With (2.13) the inverse of (2.11) can thus be expressed in the form

$$f(x) = (f'' - AF)f_o' + F\, f_o''' \qquad \text{pinned}$$

$$= F'\, f_o'' + F\, f_o''' \qquad \text{clamped} \tag{2.15}$$

Now, we apply the boundary conditions at the trailing edge $(x = s)$. For simplicity, we consider the trailing edge to be pinned if the leading edge is pinned, and clamped if the leading edge is clamped. We thus, obtain:

Pinned Edges

$$\begin{bmatrix} F'' - AF & F \\ F'''' - AF'' & F'' \end{bmatrix} \begin{pmatrix} f_o' \\ f_o''' \end{pmatrix} = 0 \tag{2.16}$$

For fixed s, this system is interpreted as a pair of equations for f_o' and f_o'''. To obtain a nontrivial solution it is necessary that the determinant, D, of the coefficient matrix vanish, or

$$D = FF'''' - F''^2 = 0 \tag{2.17}$$

This is the flutter condition within the present framework and must be solved to obtain the flutter eigenvalues. For each eigenvalue we then solve (2.16) for the ratio f_o'/f_o''' and substitute into (2.15) to obtain the flutter mode shape,

$$f(x) = \frac{F''(x)}{F''(s)} - \frac{F(x)}{F(s)} \tag{2.18}$$

Clamped Edges

We proceed in exactly the same way for clamped edges. The counterpart of the system (2.16) is

$$\begin{bmatrix} F' & F \\ F'' & F' \end{bmatrix} \begin{pmatrix} f_o'' \\ f_o''' \end{pmatrix} = 0 \tag{2.19}$$

The flutter condition and mode shape are given respectively by

$$D = FF'' - F'^2 = 0$$

$$f(x) = \frac{F'(x)}{F'(s)} - \frac{F(x)}{F(s)} \tag{2.20}$$

Step-by-Step Procedure

We can now give a detailed prescription for calculating flutter boundaries and mode shapes with the present technique.

1. Fix all but two of the parameters entering into the problem. We shall see that the reduced frequency k and the length-to-width ratio s are convenient free parameters;
2. Search the plane of free parameters for zeros of the complex determinant $D(k,s)$;
3. Evaluate the mode shape by direct evaluation of $F(x)$ at the flutter points.

The crux of the foregoing technique is the evaluation of the fundamental solution $F(x)$ (see (2.13)). The real triumph of Dr. Zeydel was to recognize that $F(x)$ and its first four derivatives could be evaluated economically on the computer. The problem of evaluating $F(x)$ is discussed in more detail in Appendix A. In the subsequent section, we proceed with some example calculations.

III. NUMERICAL EXAMPLES

The results in this section are presented for the purpose of illustrating the method of solution. They do not suffice as a comprehensive parametric survey. For this reason, comparisons with previously known results are somewhat limited.

Before we present numerical results, a few remarks on the parameters entering into the problem and used to display results are in order. The basic physical parameters are given by (2.3). The entities r_x and r_y define the ratio of membrane stresses to flexural rigidity of the panel. It is important to keep in mind that for fixed in-plane loadings of the panel, r_x and r_y vary as the inverse cube of panel thickness. Thus, solutions of the flutter condition can be expected to be quite sensitive to small changes in r_x and r_y. The dimensionless parameters R and S are essential for any panel flutter analysis. They reflect the variation in four basic physical properties:

1. Panel Material (E, ν, ρ_s)
2. Altitude (ρ_∞ , a_∞)
3. Mach Number or Speed (M or U)
4. Panel Thickness (h/b)

It is convenient to introduce three other dimensionless parameters that are functionally related to R and S , namely,

$$\mu = \frac{h}{b}\ \frac{\rho_s}{\rho_\infty} \qquad\qquad \delta = \frac{\rho_s}{\rho_\infty}\left[\frac{\rho_\infty U^2(1 - \nu^2)}{2E}\right]^{1/3}$$

$$\eta = \frac{\rho_s a_\infty}{\rho_\infty a_s} \tag{3.1}$$

where

$$a_s = \left[\frac{E}{\rho_s(1 - \nu^2)}\right]^{\frac{1}{2}} \tag{3.2}$$

is the speed of longitudinal waves (sound) in the panel material. The relationship between μ, δ, η, R and S is given by the following formulae:

$$\mu = \frac{R}{S} \qquad R = 24\,\frac{\delta^3}{\mu^2} = \mu S$$

$$\delta = \frac{R}{(24S^2)^{1/3}} \qquad S = 24\,\frac{\delta^3}{\mu^3}$$

$$\eta = \frac{1}{M}\left(\frac{S\mu^3}{12}\right)^{\frac{1}{2}} = \frac{1}{M}\sqrt{2\delta^3} \tag{3.3}$$

The dependence of these parameters on the basic physical properties is given in Table 3.1 for convenient reference.

Table 3.1 Basic Flutter Parameters

Dimensionless Parameter	Material ρ_s, a_s	Altitude ρ_∞, a_∞	Speed M	Thickness $\tau = h/b$
R	a_s^{-2}	a_∞^{2}	$12M^2$	τ^{-2}
S	$\rho_s^{-1} a_s^{-2}$	$\rho_\infty a_\infty^{2}$	$12M^2$	τ^{-3}
δ	$\rho_s^{2/3} a_s^{-2/3}$	$\rho_\infty^{-2/3} a_\infty^{2/3}$	$2^{-1/3}M^{2/3}$	--
μ	ρ_s	ρ_∞^{-1}	--	τ
η	$\rho_s a_s^{-1}$	$\rho_\infty^{-1} a_\infty$	--	--

To obtain an explicit formula for any dimensionless parameter, multiply out the dimensional parameters in the corresponding row.

The parameter η is of particular significance because it depends only on the panel material and altitude. Once η is fixed, μ becomes a direct measure of panel thickness. If η and μ are used in place of R and S or δ then the speed or Mach number becomes isolated from material properties, altitude and thickness. Therefore, the effect of Mach number changes on flutter boundaries can be treated explicitly by varying M at fixed η without inducing some unwanted change in material properties and altitude. Conversely, we can change η at fixed M and calculate the effect of a change in material or altitude.

The variation of the various parameters for different materials and altitude is shown in Tables 3.2a and b. In Table 3.2a the specific gravity, sound speed and the value of η and ρ_s/ρ_∞ at sea level are given for some different materials. Table 3.2b gives the value of the five basic parameters as a function of altitude (sea level to 50,000 ft). The data are for aluminum panels at $M = \sqrt{2}$ and a nominal panel thickness $h/b = 0.01$. We reiterate that R and S vary as the inverse square and inverse cube of the thickness, respectively, so that the numbers in the last column are not typical for all applications.

The results presented in Figs. 1 and 2 are typical results that can be calculated by the present technique. They are for pinned edge panels with $r_x = r_y = 0$. The data of Fig. 1 are obtained as follows. The parameters M, μ, η and g are fixed. Then the flutter condition (2.17) or (2.19) is expressed in the form

$$D(k,s) = D_R + iD_I = 0$$

or

$$\begin{aligned} D_R(k,s) &= 0 \\ D_I(k,s) &= 0 \end{aligned} \qquad (3.4)$$

Table 3.2a Material Properties at Sea Level

Material	Specific Gravity	Sound Speed ft/sec	η_o	ρ_s/ρ_{∞_o}
Aluminum	2.7	16,740	147.0	2200
Soft Steel	7.0	16,410	388.5	5700
Magnesium	1.8	15,100	106.2	1470
Nickel	8.5	16,320	478.0	6930
Copper	8.0	10,000	730.0	6515

Table 3.2b Variation of Panel Parameters with Altitude (τ = 0.01)

Altitude (ft)	η	δ	μ	S	R
Sea Level	147.0	29.4	22.0	48.9	1075
5000	167.8	32.0	25.5	40.7	1039
10,000	192.5	35.0	29.8	33.6	1000
20,000	256.0	39.1	41.3	22.4	922
30,000	350.0	49.5	58.9	14.4	850
40,000	522.0	65.0	89.6	9.1	817
50,000	845.0	88.2	145.0	5.6	814

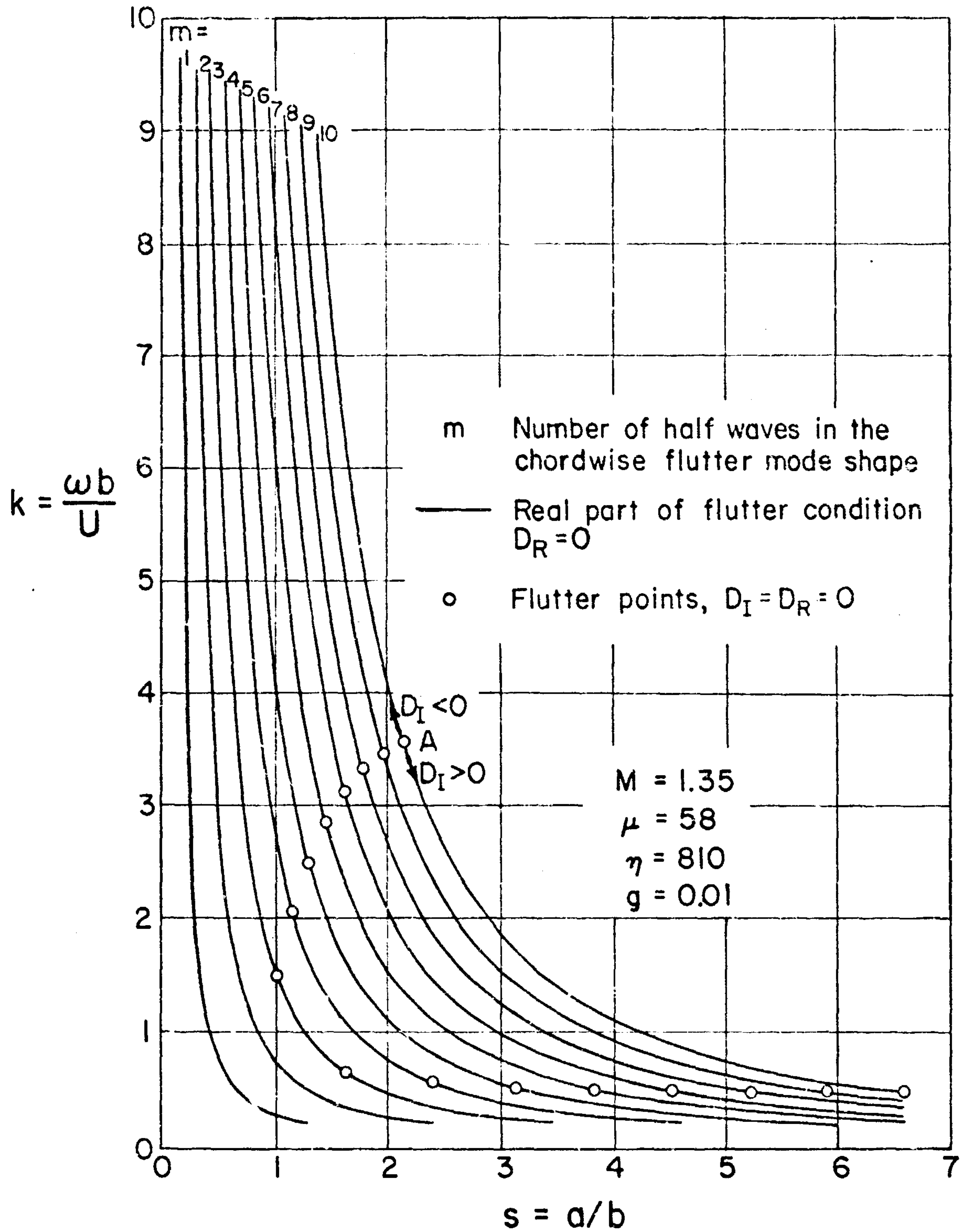

FIGURE 1. Parameter Plane and Typical Computer Calculation of Flutter Points.

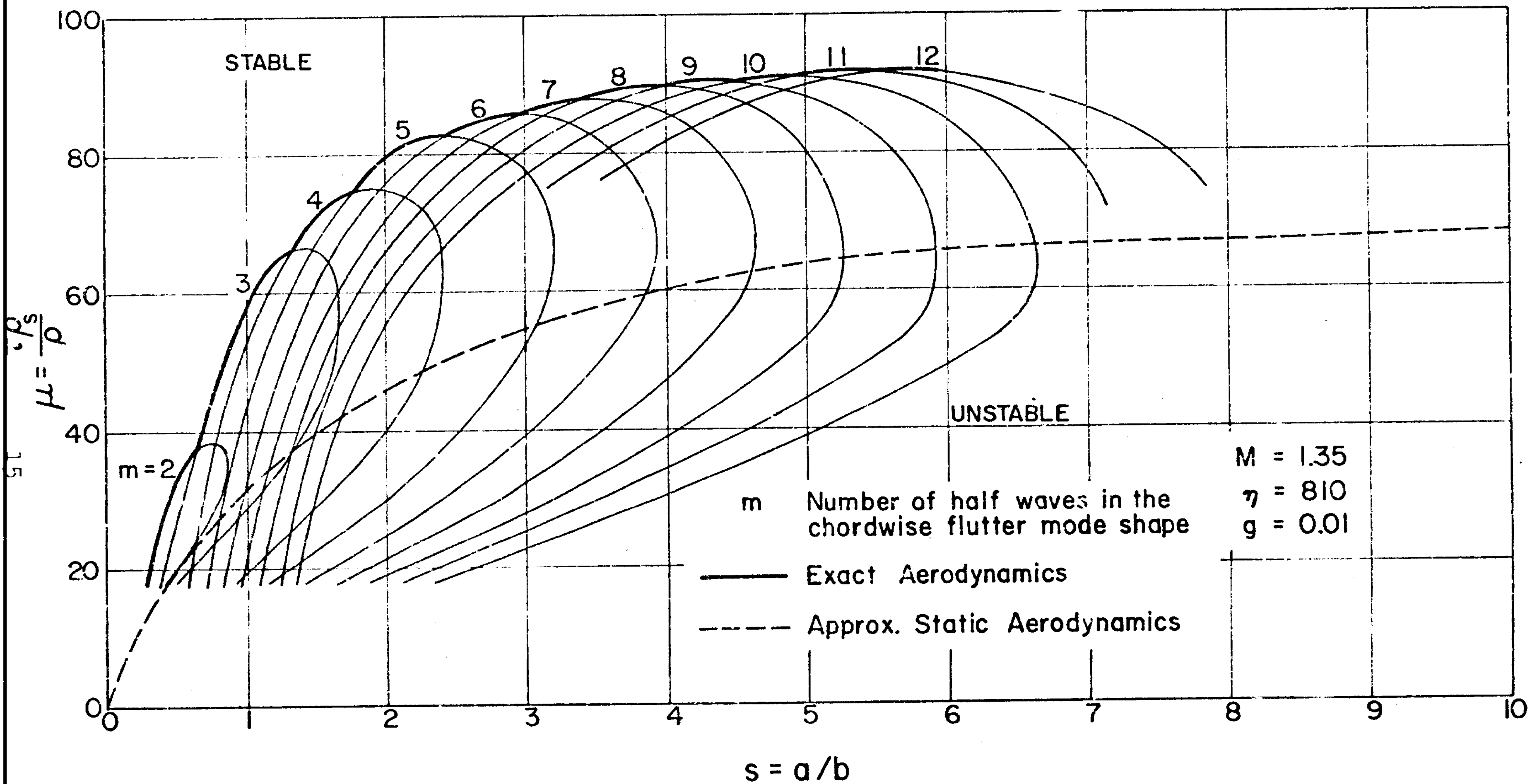

Figure 2. Typical Design Flutter Boundaries (Aluminum Panels at 50,000 ft.).

where k and s are the free parameters. Each solid curve in Fig. 1 is a locus of points where D_R is zero. The different branches (labeled $m = 1,2,\ldots,10$) reflect the number of half waves in the chordwise mode shape. For example, the curve $m = 1$ has one half wave and $m = 2$ has two half waves. In principle, there are an infinite number of possible branches, but we have arbitrarily selected $m = 10$ as a cutoff. One final remark about the solid curves in Fig. 1. They are approximated very closely by the locus of natural frequencies of the plate in a vacuum as a function of length-to-width ratio; i.e.,

$$k \cong \frac{\pi^2 \mu}{\sqrt{12}\, M\eta}\left(1 + \frac{m^2}{s^2}\right)$$

$$= 0.151\left(1 + \frac{m^2}{s^2}\right) \qquad \text{For the data of Fig. 1} \qquad (3.5)$$

This fact is of considerable value in calculating and cataloging the various branches of $D_R = 0$ on the computer.

The flutter points in Fig. 1 are obtained as follows: at consecutive points on each branch of $D_R = 0$, we compute D_I and note its algebraic sign. When a change in sign occurs, iterate for the zero value of D_I and thus obtain a flutter point (e.g., see point A in Fig. 1). In all cases that have been computed thus far, there have been either zero or two flutter points on each branch of $D_R = 0$. Also, for fixed M, μ, η and g there is always a minimum value of m for which flutter can occur. For example, the minimum number of half waves in Fig. 1 is three. These empirical observations are of considerable value in automating the calculations.

The calculations of Fig. 1 are repeated for different values of μ to obtain stability boundaries in the μ versus s plane, as shown in Fig. 2. Recall that for fixed η, μ is a direct measure of thickness so that Fig. 2 is also a set of curves of thickness to prevent flutter versus the length-to-width ratio. From Table 3.2b we note that $\eta = 810$ corresponds to an aluminum panel at approximately 50,000 ft. altitude. Each loop in Fig. 2 corresponds to one of the branches in Fig. 1 and is in itself a stability boundary. The region inside the loop is unstable. The envelope of the various loops is the stability boundary that is of engineering interest. The increase in "thickness to prevent flutter" with s is significant. It appears that the thickness tends to a limiting value with s although the gradual increase of the envelope makes this conclusion somewhat tentative. The number of half waves in the flutter mode is indicated by the numbers along the envelope. We note that this number increases indefinitely as the length of the panel is increased. The dashed line in Fig. 2 is the flutter boundary based on static aerodynamic theory. It is highly unconservative for the conditions relevant for Fig. 2. That is, a design panel thickness based on the static theory would still flutter on the basis of the more exact theory.

IV. CONCLUSIONS AND RECOMMENDATIONS

We have presented a method for calculating flutter boundaries and mode shapes for an infinite spanwise array of panels in a supersonic main stream. The principal conclusions of the study are:

1. The exact formulation of the infinite array panel flutter problem can be solved exactly by Laplace transforms without recourse to modal techniques.
2. The method of solution is particularly suited to long slender panels where modal techniques become very difficult if not impossible to apply.
3. A comparison of the time of calculation of the present method with the modal technique, for a typical case where the length-to-width ratio was ten, shows that the present method is about 200 times faster.

The present method has been developed to the point where flutter boundaries can be calculated routinely for pinned edge panels, with or without membrane stresses. Some minor problems must be resolved to calculate mode shapes efficiently. The following additional work is recommended to complete the development of the method:

1. Perfect the numerical techniques necessary to calculate flutter boundaries and mode shapes for the clamped edge case.
2. Supplement the numerical program with an asymptotic analysis of the case of long slender panels ($s \rightarrow \infty$).
3. Conduct a limited parametric survey in the interest of scrutinizing the method of calculation for numerical short cuts.
4. Incorporate the most rapid computational scheme possible into a program for calculating "stress boundaries" for the purpose of panel flutter design.

APPENDIX A
Evaluation of F(x)

Here, we describe one procedure for evaluating the fundamental function $F(x)$ that appears in Section II of this report. It should be emphasized that the success or failure of the Laplace transform technique of calculating flutter depends crucially upon the economical evaluation of $F(x)$ and its derivatives. Any short cuts that can be devised should be incorporated in subsequent applications of the procedure.

The function we must evaluate is given by equation (2.13) in the text:

$$F(x) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{px}}{\bar{D}^{+}(p)}\, dp \tag{A-1}$$

where

$$\bar{D}^{\pm}(p) = \bar{f}(p) \pm \frac{\ell(p)}{h^{\frac{1}{2}}(p)} \tag{A-2}$$

$$\bar{f}(p) = p^4 - Ap^2 + B$$

$$\ell(p) = \frac{S}{\beta}(p + ik)^2$$

$$h(p) = (p + iKM)^2 + \Gamma^2 \tag{A-3}$$

and the constants, A, B, S, etc., are defined in the text. Define the quantity

$$\Delta(p) = h(p)\, \bar{D}^{+}(p)\, \bar{D}^{-}(p) \tag{A-4}$$

which is a tenth degree, polynomial in p with roots p_n, n 1,2,...,10 , that are solutions of $\Delta(p) = 0$. Except in certain limiting cases (e.g., vanishing aerodynamic forces) these roots are distinct, so that

$$\frac{1}{\Delta(p)} = \sum_{n=1}^{10} \frac{1}{\Delta'(p_n)(p - p_n)} \tag{A-5}$$

where

$$\Delta'(p) = \frac{d}{dp}\Delta(p) \tag{A-6}$$

To render the function $h^{\frac{1}{2}}(p)$ single valued, we cut the complex p plane between the branch points $(p = -iKM \pm i\Gamma)$ as shown in Fig. A-1. The zeroes of $\Delta(p)$ and the path of integration used to evaluate $F(x)$ are also shown for convenient reference. We remark that the pairs of zeroes that lie close to and far out on the axes (e.g., p_1, p_6 or p_2, p_7) correspond to the free vibration modes of the panel when all aerodynamic forces vanish. The zeroes p_5 and p_{10} correspond to the travelling wave solutions of an infinitely long panel. The essence of panel flutter is tied up with the location of these physically meaningful singularities.

Now we can write $F(x)$ in the form

$$F(x) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{px}}{\Delta(p)}\left[h(p)\overline{f}(p) - \ell(p)h^{\frac{1}{2}}(p)\right] dp$$

$$= \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{px}h(p)\overline{f}(p)}{\Delta(p)} - \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} e^{px}\,\frac{h(p)\ell(p)}{\Delta(p)}\cdot\frac{dp}{h^{\frac{1}{2}}(p)} \tag{A-7}$$

and invert each term on the right-hand side separately. Noting (A-5) we observe that the first term is given simply by the sum of the residues at the ten poles in Fig. A-1. The second term is most conveniently expressed as a convolution of the inverse transforms of $\frac{h(p)\ell(p)}{\Delta(p)}$ and $\frac{1}{h^{\frac{1}{2}}(p)}$ separately. The final result can be expressed concisely in the following form:

$$F(x) = \sum_{n=1}^{10} F_n(x) \qquad \text{(A-8)}$$

where

$$F_n(x) = e^{p_n x}\left(A_n - B_n G_n(x)\right)$$

$$G_n(x) = \int_0^x e^{-(p_n + iKM)\xi}\, J_0(\Gamma\xi)\,d\xi$$

$$A_n = \frac{h(p_n)\bar{f}(p_n)}{\Delta'(p_n)} \quad , \quad B_n = \frac{h(p_n)\ell(p_n)}{\Delta'(p_n)} \qquad \text{(A-9)}$$

To formulate the flutter condition we must calculate the first four derivatives of $F(x)$. These can be calculated formally by differentiating (A-8) and the auxilliary definitions (A-9). However, a significant simplification that is crucial for the efficient evaluation of the derivatives is based on the following result:

$$\int_C \frac{p^m}{\Delta(p)}\,dp = 0 \qquad m = 1,2,\ldots,8 \qquad \text{(A-10)}$$

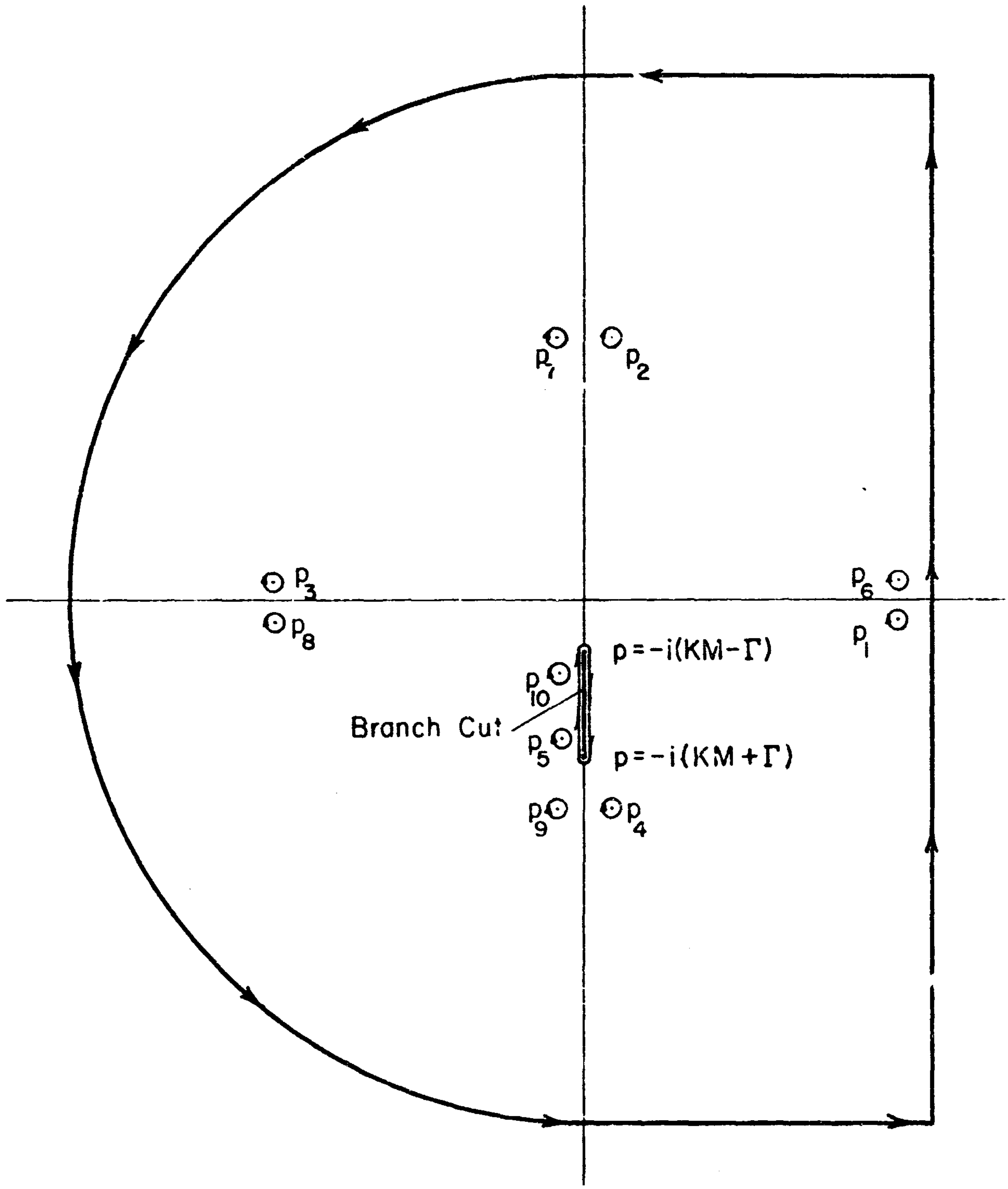

FIGURE A-1. Singularities in the Complex p-plane and Path of Integration for the Inverse Laplace Transform.

where C is any closed curve that contains all of the poles of the integrand. The result is easily proved by reversing the path of integration and noting that there is no residue at infinity. It follows at once from (A-10) that

$$\sum_{n=1}^{10} \frac{p_n^{\,m}}{\Delta'(p_n)} = 0 \qquad m = 0,1,2,\ldots,8 \tag{A-11}$$

and therefore

$$\sum_{n=1}^{10} A_n p_n^{\,m} = 0 \qquad m = 0,1,2$$

$$\sum_{m=1}^{10} B_n p_n^{\,m} = 0 \qquad m = 0,1,2,3,4 \tag{A-12}$$

With the foregoing results, it is readily verified that the first four derivatives of $F(x)$ can be written in the convenient form

$$F^{(m)}(x) = \sum_{n=1}^{10} p_n^{\,m} F_n(x) \qquad m = 0,1,2,3,4 \tag{A-13}$$

If we substitute the last result into the flutter condition and mode shape for pinned and clamped edges (recall (2.17), (2.18), and (2.19)), we obtain the following results:

<u>Pinned Edges</u>

$$D = \sum_{n=1}^{9} \sum_{m=n+1}^{10} F_n(s)F_m(s)(p_n^{\,2} - p_m^{\,2})^2 = 0 \qquad \text{Flutter Condition}$$

$$f^{(m)}(x) = \frac{\sum_{n=1}^{10} p_n^{m+2} F_n(x)}{\sum_{n=1}^{10} p_n^2 F_n(s)} - \frac{\sum_{n=1}^{10} p_n^{m} F_n(x)}{\sum_{n=1}^{10} p_n^2 F_n(s)} \qquad m = 0,1,2$$

Mode Shape and Derivatives

(A-14)

Clamped Edges

$$D = \sum_{n=1}^{9} \sum_{m=n+1}^{10} F_n(s) F_m(s) (p_n - p_m)^2 = 0$$

$$f^{(m)}(x) = \frac{\sum_{n=1}^{10} p_n^{m+1} F_n(x)}{\sum_{n=1}^{10} p_n F_n(s)} - \frac{\sum_{n=1}^{10} p_n^{m} F_n(x)}{\sum_{n=1}^{10} F_n(s)} \qquad m = 0,1,2$$

(A-15)

The entire problem of evaluating $F(x)$, flutter boundaries, and mode shapes has been reduced to the evaluation of the ten functions $F_n(x)$. The latter are simply the sum of an exponential and the convolution of an exponential with a Bessel function. Any standard routine can be used for the evaluation of these functions.

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